

# LOCAL POLYNOMIAL RECONSTRUCTION OF INTENSITY DATA AS BASIS OF DETECTING HOMOLOGOUS POINTS AND CONTOURS WITH SUBPIXEL ACCURACY APPLIED ON IMAGER 5003

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## ABSTRACT:

Laserscanners are used more and more as surveying instruments for various applications in industry as well as in architectural heritage conservation. With the advance of high precision systems, capable of working in most real world environments under a variety of conditions, numerous applications have opened up. The developed IMAGER 5003 is a state-of-the-art, high precision, high speed laser scanner that provides accurate 3D measurements. So high accuracy in extracting features is most important. The IMAGER 5003 measures both, range and reflectance (=intensity) images at the same time. This paper deals with an algorithm for detecting corners and contours at a subpixel accuracy: The first part of the paper explains how features in the reflectance image can be extracted by local polynomial fitting. Based on this, the second part focuses on the precise detection of contours and corners. The paper concludes with results by applying the algorithm to scanned data in real environment which shows its robustness.

## 1. INTRODUCTION AND MOTIVATION

Detecting contours, corners and edges are important and well known in 2D image Analysis. Corners and edges should be located precisely, true corners and not false corners should be detected and last but not least, the method must be stable, especially to noise or artefacts.

Beside the 2D intensity information the laser scanner Imager 5003 has in addition 3D range information (see [Frö02], [Ste02], [Hae01], [Hei01]). The system has different scanning modes, which differ in spatial point distance. It can be selected adapted to the requirements and application between Super High Resolution (20000 Pixel per 360° horizontally and vertically) and Preview (1250 Pixel per 360° horizontally and vertically) mode. Both information – reflectance and 3D geometry – are corresponding by each pixel. So by extracting features in an accurate way, the combination of image processing methods and 3D geometric information is possible.

In this paper we present an algorithm which detects corners and edges by reconstructing the reflectance image (=intensity data) of Imager 5003 by using polynomials. So first is shown, how the reflectance image of Imager 5003 can be reconstructed locally by polynomials of degree less or equal than two. The second part of the paper deals with the curvature (as a characteristic of the reconstructed polynomial) and how it can be used as a classification criteria for corners and edges. Examples are presented in the third part. Furthermore some aspects of a fast computation of the method are explained.

Like in all measurement systems, the reflectance and the range data of the IMAGER 5003 is more or less noisy. But in addition, the developed method must be stable to artefacts: Depending on the mode, the spatial point distance decreases, which results in an increase of quantisation effects. So on the one hand, the reconstruction of the reflectance image is a basic

motivation, but on the other hand however, the developed method must be fast enough to be practical in industrial applications.



Figure 1: The figure shows the Laser scanner IMAGER 5003.

Based on this ideas, polynomials were chosen for solving this task, which is shown in the next section.

## 2. POLYNOMIAL RECONSTRUCTION AND FEATURE EXTRACTION

### 2.1 Local Image Reconstruction

Denote  $D := \{0..n\} \times \{0..m\}$  the image square. Let

$$g : \begin{cases} D \rightarrow \{0..N\} \\ (u, v) \mapsto g(u, v) \end{cases}$$

be the reflectance image. Be  $(i, j)$  optional but fixed a co-ordinate in  $\mathbf{g}$ . Denote

$$N_{k,l}(i, j) := \{(i - k, j - l), \dots, (i + k, j + l)\}$$

the  $(2k + 1) \cdot (2l + 1) - 1$  neighbourhood around the pixel  $(i, j)$ . Now a polynomial

$$p : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$$

of degree less or equal than two is searched, which should minimise

$$\sum_{(u,v) \in N_{k,l}} (p(u, v) - g(u, v))^2 \rightarrow \min. \quad (**)$$

This optimisation problem can be solved with a linear least squares approach: To show this, choose e.g. the monomials  $\{1, x, y, x^2, y^2, xy\}$  as basis for  $p$  and set  $v_{x,y} := (1, x, y, x^2, y^2, xy)$  and  $\mathbf{g}_{i,j} := \mathbf{g}(i, j)$  to simplify notation. Denote  $\|x\|_2 := \sqrt{x^t x}$  the euclidean distance and set

$$\mathbf{b} := (\mathbf{g}_{i-u, j-v}, \dots, \mathbf{g}_{i, j}, \dots, \mathbf{g}_{i+u, j+v})^t$$

and

$$\mathbf{G} := (v_{i-u, j-v}, \dots, v_{i, j}, \dots, v_{i+u, j+v})^t.$$

Then  $\det(\mathbf{G}) \neq 0$  and the in  $(*)$  formulated minimisation problem can be rewritten as

$$\|\mathbf{G} \cdot \mathbf{a}_0 - \mathbf{b}\|_2 = \min_{\mathbf{a} \in \mathbb{R}^6} \|\mathbf{G} \cdot \mathbf{a} - \mathbf{b}\|_2. \quad (**)$$

This linear least squares problem is well known in literature and can now be solved with a standard method, for example the householder algorithm.

### Implementation

From a computational point of view it is better to transform the image co-ordinate system into a local co-ordinate system with the pixel  $(i, j)$  as origin: The matrix  $\mathbf{G}$  and in particular its pseudo inverse  $\mathbf{G}^+$  will then be identical for each neighbourhood  $N_{k,l} \subset D$ . Thus the in  $(**)$  reformulated minimisation problem simplifies to

$$\mathbf{a}_0 = \mathbf{G}^+ \cdot \mathbf{b}.$$

## 2.2 Features for Contours and Corners

If you focus on the grayvalues in the neighbourhood around a pixel  $(i, j)$ , then the change in the graylevels is the most significant criteria for classifying contours and corners: If you move along a contour, then the grayvalues will not change significantly. But if you move along the direction which is orthogonal to the 'contour-line', then the values will change a lot. Vice versa if the pixel  $(i, j)$  is element of a corner: Now the change between the grayvalues along each direction is significant.

To express this now in a more mathematical sense, each pixel is identified with its feature vector  $\mathbf{a}_0 \in \mathbb{R}^6$  characterising the polynomial  $p$  as described above. Now it is the curvature of  $p$ , which can be taken as a criteria of the change in the grayvalues: As is generally known, the curvature of a polynomial  $p$  can be calculated by differentiating  $p$  two times: The Hessian  $H_p$  of the second partial derivatives is then defined through

$$H_p := \begin{pmatrix} \frac{d^2}{dx \cdot dx} p & \frac{d^2}{dx \cdot dy} p \\ \frac{d^2}{dy \cdot dx} p & \frac{d^2}{dy \cdot dy} p \end{pmatrix}$$

and short

$$H_p := \begin{pmatrix} d_{xx} & d_{xy} \\ d_{yx} & d_{yy} \end{pmatrix}.$$

The maximal and minimal curvature of  $p$  correspond to the two eigenvalues  $\lambda_{\max}$  and  $\lambda_{\min}$  of  $H_p$ : They are real, as  $H_p$  is symmetric and can be calculated by solving the zeros of the characteristic polynomial  $\gamma$  of  $H_p$  through

$$\gamma_p := \det \left( H_p - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right) = 0$$

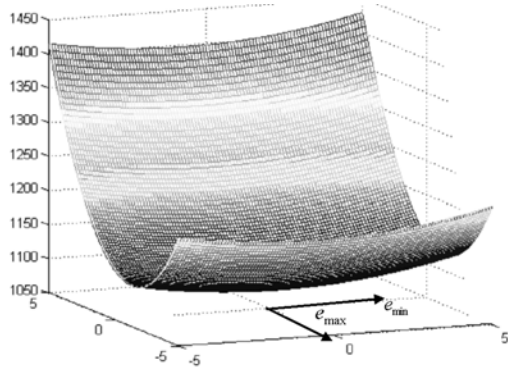
and hence

$$\lambda_{1,2} = \frac{(d_{xx} + d_{yy}) \pm \sqrt{(d_{xx} + d_{yy})^2 - 4(d_{xx} d_{yy} - d_{xy}^2)}}{2}$$

and hence

$$\lambda_{\max} := \max \{ |\lambda_1|, |\lambda_2| \}$$

$$\lambda_{\min} := \min \{ |\lambda_1|, |\lambda_2| \}.$$



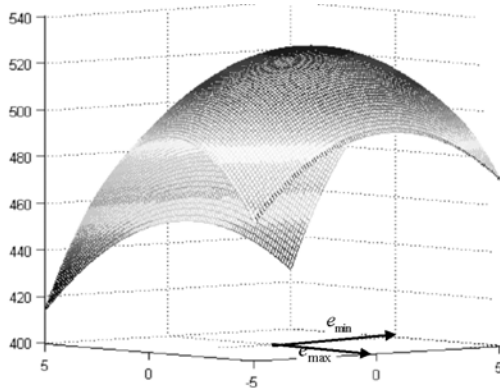
**Figure 2:** The figure shows a polynomial  $p$  of degree 2 which reconstructed the neighbourhood around a line. Also marked are the eigenvectors of the Hessian of  $p$ . The eigenvalue corresponding to the eigenvector  $e_{\min}$  will be small, the eigenvalue corresponding to the eigenvector  $e_{\max}$  will be large.

Similar to the in [Har88] from Harris and Stephens introduced criteria, a classification for contours and corners can now be described as follows: Set

$$\xi_p := \alpha \lambda_1 \lambda_2 - (\lambda_1 + \lambda_2) \quad (***)$$

for an adequate  $\alpha \in \mathbb{R}^+$ . Let  $T_1$  and  $T_2$  be thresholds known from experience; then the pixel  $(i, j)$  is element of a contour (see Figure 2) if

$$(P.1) \quad (i, j) \in \text{Contour} \Leftrightarrow T_1 < \left| \xi_p \right| .$$



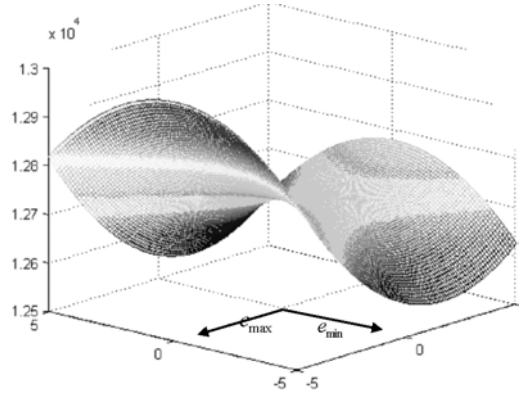
**Figure 3:** The figure shows a polynomial  $p$  of degree 2 which reconstructed the neighbourhood around a corner. Also marked are the eigenvectors of the Hessian of  $p$ . Either both eigenvalues are positive or negative.

The Pixel  $(i, j)$  is element of a corner (see Figure 3), if

$$(P.2) \quad (i, j) \in \text{Corner} \Leftrightarrow T_2 < \xi_p .$$

And additional it can be given a classification for saddles (see Figure 4) through:

$$(P.3) \quad (i, j) \in \text{Saddle} \Leftrightarrow \xi_p < -T_2$$



**Figure 4:** The figure shows a polynomial  $p$  of degree 2 which reconstructed the neighbourhood around the extremum of a saddle. Also marked are the eigenvectors of the Hessian of  $p$ . One eigenvalue will be positive, one negative.

### 2.3 Subpixel Extraction

In the last section, a feature was described, which enables the identification of a pixel  $(i, j)$  as being element of a contour, corner or saddle. In the next paragraph it will be shown, how contour points can be calculated with subpixel accuracy.

#### Contour Points

Let  $S \subseteq D$  be the subset of the image square  $D$ , which holds property (P.1) and let  $F : D \rightarrow \mathbb{R}$  be the feature image, which maps each pixel onto the in (\*\*\*) described feature value: The closer the pixel  $(i, j)$  is to the skeleton, the bigger the function value  $F(i, j)$  will be.

Based on this observation, the idea is now, to reconstruct the feature image  $F$  in the subset  $S$  with the algorithm shown in 2.1 once again, and to calculate the extremum with subpixel accuracy. Thus let  $(i, j)$  be a pixel in the subset  $S$ , and let  $q$  be the polynomial, which reconstructed  $F$  in the neighbourhood around  $(i, j)$ . Let  $w_{\max}$  be the eigenvector of the hessian  $H_q$  of  $q$ , which corresponds to the bigger eigenvalue  $\eta_{\max}$  and let  $w_{\min}$  be the eigenvector corresponding to the smaller eigenvalue  $\eta_{\min}$  respectively. Denote

$$h := pr_{w_{\max}}(q)$$

the projection from  $q$  onto  $w_{\max}$ . Then  $h : \mathbb{R} \rightarrow \mathbb{R}$  is a polynomial of degree 2, and its extremum is given through the first derivation

$$\frac{d}{dx} h(x) = \frac{d}{dx} (a_0 + a_1 x + a_2 x^2)$$

and thus

$$t_{\max} = -\frac{a_1}{2 \cdot a_2} .$$

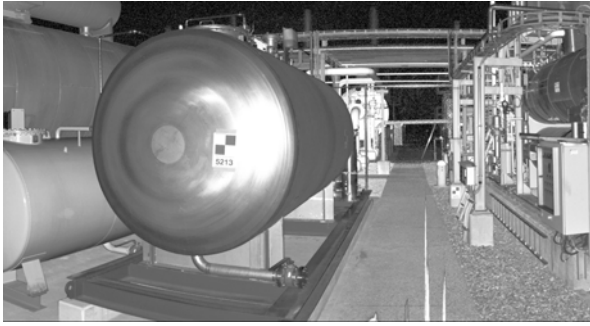
Now the subpixel value on the skeleton can be calculated through

$$(i', j') = (i, j) + w_{\max} \cdot t_{\max}.$$

*note:* Moreover the direction of the contour is known, as you know the eigenvector of the smaller eigenvalue. This could be a very helpful information for further image processing algorithms.

### Corners and Saddles

In this paragraph it will be shown, how a corner and the extremum of a saddle can be calculated: Let now  $S \in D$  be the subset of the image square  $D$ , which holds property (P.2) (or property (P.3) for a saddle respectively) and let  $F$  be the feature image, which maps each pixel onto its feature value  $\xi_p$ . Again let  $(i, j) \in S$  be a pixel in the subset  $S$ , and let  $q$  be the polynomial, which reconstructed  $F$  in the neighbourhood around  $(i, j)$  (similar to the algorithm shown in section 2.1). Let  $w_{\max}$  be the eigenvector of the hessian  $H_q$  of  $q$ , which corresponds to the bigger eigenvalue  $\eta_{\max}$  and let  $w_{\min}$  be the eigenvector corresponding to the smaller eigenvalue  $\eta_{\min}$ .



**Figure 4:** The Figure shows the reflectance image of a typical scan in an industrial environment.

Denote

$$h_{\max} := pr_{w_{\max}}(q)$$

the projection from  $q$  onto  $w_{\max}$  and denote

$$h_{\min} := pr_{w_{\min}}(q)$$

the projection from  $q$  onto  $w_{\min}$ . Then  $h_{\min} : IR \rightarrow IR$  and  $h_{\max} : IR \rightarrow IR$  are polynomials of degree 2, and their extremums  $t_1$  and  $t_2$  are given through the first derivation (which can be calculated like demonstrated in the last paragraph). The subpixel value of a corner or saddle can now be calculated through

$$(i', j') = (i, j) + w_{\max} \cdot t_1 + w_{\min} \cdot t_2.$$

## 3. APPLICATIONS

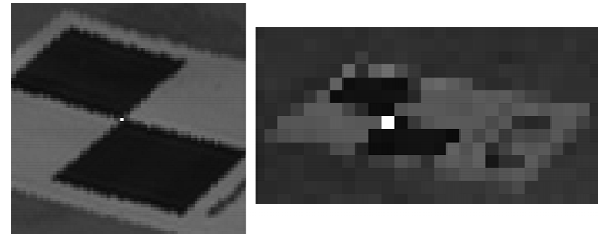
In this section applications for the described methods are demonstrated, based on real environmental data sets. Fig. 4 shows a section of a reflectance image taken in typical industrial environments. Targets are used in this environment to reference multiple images with respect to each other. We present two different applications which are used in real environments, namely detecting the centre of a target as well as finding the extremum of a corner.



**Figure 5:** The figure shows the reflectance image of a typical scan in real environment.

### 3.1 Finding the Centre of a Target

In real environments targets are used in order to register many individual scans together. Therefore a crucial element for a target finder is to locate targets in real environments with a very high accuracy. As already small deviations in localisation may cause very big errors in large and extended environments, the detection of targets in subpixel accuracy is necessary. In these environments targets can not be fixed always perpendicular to the scanning system (IMAGER 5003). Furthermore it is of great interest that the image processing steps are robust, first to spatial resolution and second to orientation of the target surface relative to the scanner itself.



**Figure 6:** The figure shows the result of the algorithm as used as target finder: The marked point is the extremum of a saddle. *Left image:* The scan was taken with a spatial resolution of 10000 Pixel per 360 degree horizontally and vertically. The range is approx. 4 m. *Right image:* The scan was taken with a spatial resolution of 1250 Pixel per 360 degree horizontally and vertically. The range is approx. 4 m.

The algorithm itself, is now straight forward by using the introduced methods: For this, let  $S \subset D$  be the subset of the image square, in which the center of the target should be

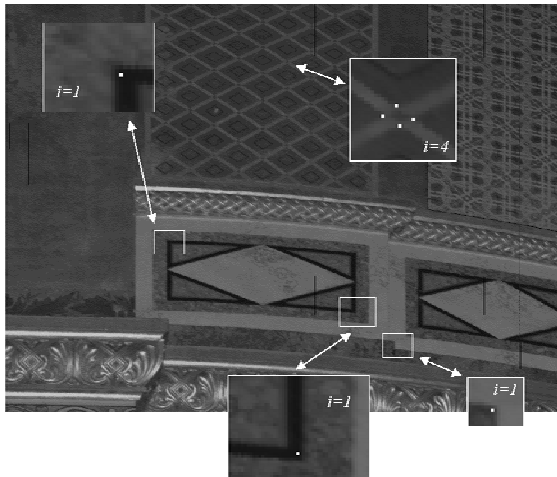
detected. Denote again  $F : S \rightarrow \mathbb{R}$  the feature image. Let  $(x,y)$  be the pixel in  $S$  for which  $F$  is minimal. Then the subpixel extremum of the saddle is calculated as described in Section 2.3.

In Fig. 6 we show the results for two different targets, varied in range, spatial resolution and orientation. The results show a robust and subpixel accuracy, even with extreme variations. These results underline that the motivation of using a polynomial approach is more suited than standard methods for target finding like convolution approaches etc.

Fig. 5 shows the application of the developed target finder in real environments like a forest.

### 3.2 Finding the Extremum of a Corner

In most industrial applications targets are usually the best and fastest method to register many individual scans together. However, targets are undesired in some 3D reconstruction applications: In cultural heritage conservation for instance, a combination with RGB colour information is requested often. It would be incommode, if in the reconstructed model targets would appear.



**Figure 7:** The figure shows the result of the algorithm used as corner detector: The subpixel extremum are calculated for the  $i$  biggest values of the feature image  $F$  in the selected area. The image was taken with a High Resolution scan (10000 Pixel per 360 degree horizontally and vertically).

The algorithm for finding the centre of a corner is similar to the target finder shown above. But contrarily to targets, more than one corner might be wished to be detected: So let  $S \subset D$  again be the subset of the image square, in which corners should be detected. Denote again  $F : S \rightarrow \mathbb{R}$  the feature image. Let

$$M_i = \{(x_1, y_1), \dots, (x_i, y_i)\} \subset S$$

be the set of pixels ( $i$  indicates the amount of corners you wish to detect in  $S$ ) for which  $F$  is bigger than for each other  $(u, v) \in S/M_i$ . Then the subpixel extremum of the corners can be calculated like shown in Section 2.3.

Figure 7 shows examples taken of a facade of a building, which shows again the robustness of the method.

## 4. DISCUSSION AND OUTVIEW

In this paper we presented a method for Imager 5003, which reconstructs the local neighbourhood of a pixel by polynomials, and thereby enables the classification of contours, saddles and corners as well as its subpixel detection. The method works quite well for detecting corners and the extremum of saddles. In further studies, it will be worked out, if the algorithm is stable under different perspective variations. Furthermore this has to be tested to further experiments, especially to prove stability.

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